STUDIES ON MIXING. XXXIV.*

A RELATION BETWEEN THE RATE OF HOMOGENATION OF MISCIBLE LIQUIDS AND THE PUMPING CAPACITY OF A TURBINE MIXER

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A relation is derived for determining the time necessary to achieve a required degree of homogeneity of the charge for a mechanically mixed system with a turbine mixer and radial beffles under the turbulent regime of flow of the charge. The pumping capacity of the mixer is taken as a parameter. A sufficient agreement is found between the theoretical and experimentally determined time of homogenation as long as the required degree of homogeneity is less than 0·10.

Homogenation of miscible liquids by mechanical mixing is a process often acompanying technological operations in liquid medium. A knowledge of the rate of homogenation is then usually mandatory in judging the suitability of a given mixing equipment for a given operation. The time course of homogenation of miscible liquids by mechanical mixer has been described in the literature several times both theoretically[1-5] and experimentally[1,2,6]. The result of all cited papers is, as a rule, a dependence of dimensionless time of homogenation on geometry parameters of the system for a requested degree of homogeneity of the charge.

Let us consider a newtonian liquid mixed by a standard six-blade turbine mixer located in the axis of a cylindrical vessel with four radial baffles at the wall[7]. The mixer causes vigorous turbulent flow of liquid, whose measure may be, for example, the pumping capacity of the mixer. A small volume (negligible in comparison with the volume of the charge V) of a tracer of concentration c_0, different from c_0 (usually higher), is added to the system of spatially homogeneous concentration c_0', in which a solid phase is being dissolved. Two volumes, V_1 and V_2, such that they fill the whole volume of the charge, are delimited in the system and the sample ΔV of the tracer is added to the volume V_1. The volumes V_1 and V_2 are chosen in such a manner that the stream of liquid (the pumping capacity of the mixer) driven by the mixer passes through both of them. Let us introduce following assumptions: *1.* The pumping capacity $\dot{V}$, as well as the volumes V_1 and V_2 are constants in time. *2.* The initial (c_0) and the final (c_k) concentration of the dissolved substance in the charge are uniform in the whole volume. *3.* Mixing in volumes V_1 and V_2 is ideal.

Differential mass balance of the time course of equalization of concentrations in the volumes V_1 and V_2, assuming validity of the above listed assumptions may be written as[3,4]

$$V_1\, \mathrm{d}c_1/\mathrm{d}\Theta = \dot{V}(c_2 - c_1)\,, \tag{1}$$

$$V_2\, \mathrm{d}c_2/\mathrm{d}\Theta = \dot{V}(c_1 - c_2)\,, \tag{2}$$

and the initial conditions are

$$\lim_{\theta \to 0_+} c_1 = c_0 + \Delta V c_0'/V\,, \tag{3a}$$

$$\lim_{\theta \to 0_+} c_2 = c_0 \tag{3b}$$

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On solving the set of differential equations (*1*) and (*2*) by the Laplace transform we obtain

$$\frac{c_1 - c_0}{c_k - c_0} = 1 + \frac{V_2}{V_1} \exp\left(- \frac{V}{V_1 V_2} \dot{V} \Theta\right), \tag{4}$$

$$\frac{c_2 - c_0}{c_k - c_0} = 1 - \exp\left(- \frac{V}{V_1 V_2} \dot{V} \Theta\right). \tag{5}$$

While the over-all balance requires that

$$V_1 + V_2 = V \tag{6}$$

and

$$c_k - c_0 = \Delta V c_0' / V. \tag{7}$$

Eqs (*4*) and (*5*) may be rearranged so as to give expressions for the earlier[1] defined degree of homogeneity. These are

$$X_1 = \frac{|c_1 - c_k|}{c_k - c_0} = \frac{V_2}{V_1} \exp\left(- \frac{V}{V_1 V_2} \dot{V} \Theta\right), \tag{8}$$

$$X_2 = \frac{c_2 - c_k}{c_k - c_0} = \exp\left(- \frac{V}{V_1 V_2} \dot{V} \Theta\right). \tag{9}$$

Eqs (*8*) and (*9*) express the dependence of the degree of homogeneity for volume V_1 to which the tracer was added, or for the volume V_2 into which the tracer is being transported from the volume V_1. For the values of the degree of homogeneity it is stipulated by the boundary condition that

$$\lim_{\theta \to 0_+} X_1 = V_2 / V_1 , \qquad \lim_{\theta \to \infty} X_1 = 0 , \tag{10a}$$

$$\lim_{\theta \to 0_+} X_2 = 1 , \qquad \lim_{\theta \to \infty} X_2 = 0 . \tag{10b}$$

Let us consider now two situations important from practical point of view: In the first case the volume V_1 is regarded as a space occupied by the rotating mixer[4], while the volume V_2 fills the rest of the charge. The volume occupied by the mixer is further assumed to be negligible in comparison with the total volume of the charge. Obviously, we have that

$$\lim_{\substack{\theta \to 0_+ \\ V_1 \ll V}} X_1 = 8 , \qquad \lim_{\substack{\theta - \infty \\ V_1 \ll V}} X_1 = 0 , \tag{11a}$$

$$\lim_{\substack{\theta \to 0_+ \\ V_1 \ll V}} X_2 = 1 , \qquad \lim_{\substack{\theta - \infty \\ V_1 \ll V}} X_2 = 0 , \tag{11b}$$

i.e. the degree of homogeneity in the volume V_1 attains very high values shortly after addition of the tracer. The degree of homogeneity in the volume V_2 characterizes then the concentration in practically the whole volume of the charge. In the second case we consider the volumes V_1 and V_2 equal, while the mixer is in V_1. This volume represents then a cylinder surrounded by the volume V_2 (hollow cylinder) so that the stream driven by the mixer (the pumping capacity of the mixer) passes through both of these volumes:

$$V_1 = V_2 = V/2 \tag{12}$$

and

$$X = \frac{|c - c_k|}{c_k - c_0} = X_1 = X_2 , \quad [V_1 = V_2] . \tag{13}$$

The degree of homogeneity is then uniform in the whole volume of the mixed charge for a given time Θ. This fact has been verified experimentally[1] for the above described system. From Eqs (*8*), (*9*), (*12*) and (*13*) follows the relation

$$\log (1\cdot0/X) = 4\dot{V}\Theta/(2\cdot303\,V) , \quad [V_1 = V_2] . \tag{14}$$

Following relationship for the dimensionless time of homogenation[1] follows from experimental investigation of the time course of homogenation in a system with radial baffles and a turbine impeller located with its plane of symmetry half-way ($h_2/D = 0\cdot5$) between the bottom an the liquid level.

$$n\Theta = 0\cdot905(D/d)^{2,57} \log (2\cdot0/X) , \quad [\mathrm{Re} > 1\cdot0 \,.\, 10^4] . \tag{15}$$

Since[7]

$$\dot{V} = K_p n d^3 , \quad [\mathrm{Re} > 1\cdot0 \,.\, 10^4] \tag{16}$$

and for given experimental set-up $H = D$, we have for V

$$V = \pi D^3/4 \tag{17}$$

and then by substituting into Eq. (*14*) from (*16*) and (*17*) and after some arrangement we obtain

$$n\Theta = (0\cdot425/K_p)\,(D/d)^3 \log (1\cdot0/X) , \quad V_1 = V_2 \; \mathrm{Re} > 1\cdot0 \,.\, 10^4 . \tag{18}$$

TABLE I

Comparison of the Dimensionless Time of Homogenation Calculated from the Model with that Found Experimentally for Turbine Mixer $h_2/D = 0\cdot5$

Found Experimentally for Turbine Mixer $h_2/D = 0\cdot5$ Re $> 1\cdot0 \,.\, 10^4$.

	$D/d = 3$	$K_p = 0\cdot76$	
X	0·1	0·05	0·02
$(n\Theta)_{calc}$	161	21·0	27·4
$(n\Theta)_{exp}$	19·8	24·5	30·6
	$D/d = 4$	$K_p = 0\cdot7$	
X	0·1	0·05	0·02
$(n\Theta)_{calc}$	41·3	53·8	70·2
$(n\Theta)_{calc}$	41·6	51·2	63·8

Table I gives a comparison of the dimensionless time of homogenation calculated from Eqs (*15*) and (*18*) for the degree of homogeneity 0·10, 0·05, 0·02. The comparison is made for two dimensions of the mixer, for which the quantity K_p (flow rate criterion) was determined from the measurement of the velocity field near the outer edge of rotating blades* of the mixer[7]. The table indicates a good agreement of dimensionless times of homogenation calculated from Eq. (*18*), *i.e.* from the model, with the values $(n\Theta)_{exp}$ calculated from experimental results of the time course of homogenation, *i.e.* Eq. (*15*). The average relative deviation of corresponding pairs in Table I amounts to 11·5% which is better than the average error of the experimentally determined time of homogenation[1]. For higher degree of homogeneity than 0·1, however, the disagreement between $(n\Theta)_{exp}$ and $(n\Theta)_{calc}$ is so marked that the adopted model of homogenation can no longer be ragarded as correct. That may be attributed to invalidity of assumption 3 since at the beginning of homogenation the added tracer can be assumed to be uniformly dispersed neither in the volume V_1 nor V_2. For lower degree of homogeneity, usually requested at mixing of two miscible liquids, the proposed model can be used for description of the process. The time of homogenation of miscible liquids mixed by the turbine mixer under the turbulent regime thus can be calculated for the degree of homogeneity less than 0·10 from known pumping effects of the mixer, the geometry parameters of the system and the frequency of revolution, n, of the mixer, provided that the volumes V_1 and V_2 are chosen so as to be both passed through by the pumping capacity of the mixer.

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LIST OF SYMBOLS

c	concentration of dissolved substance in charge (kg m^{-3})
c'	concentration of tracer in sample (kg m^{-3})
D	diameter of vessel (m)
d	diameter of mixer (m)
H	clear liquid height (m)
h_2	height of the plane of mixer over bottom (m)
n	frequency of revolution (s^{-1})
V	volume of charge (m^3)
ΔV	volume of tracer (m^3)$_{mixer}$
$\dot{V}$	pumping capacity of mixer (m^3 s^{-1})
Θ	time of homogenation (s)
ν	kinematic viscosity (m^2 s^{-1})
$X = \dfrac{\lvert c - c_k\rvert}{c_k - c_0}$	degree of homogeneity
$K_p = \dot{V}/nd^3$	flow rate criterion
$\mathrm{Re} = nd^2/\nu$	Reynolds number

* The relative height of the impeller over the bottom of the vessel was 1/3 in these experiments. The pumping capacity of the mixer, however, does not depend[7-9] on the quantity h_2/D in the range $h_2/D \in \langle 1/3; 1/2 \rangle$ and the mentioned results may thus be used for determining K_p. Moreover, from the cited paper follows that in the situation when V_1 is surrounded by V_2, the pumping capacity of the mixer passes through both of them.

Subscripts

exp	calculated from experimental results
calc	calculated from model
0	initial value
k	final value
1	related to the volume V_1 to which the sample of tracer is added
2	related to the volume V_2 to which no tracer is added

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